Chapter 5

Strategic interactions in the labor market, self-esteem motivations and socio-demographic disparities

In the previous chapter, we have posited a theory of socio-demographic disparities in the labor market based on a special kind of occupational segregation. From the characterization of a subset of selective (upon socio-demographic criteria) jobs within the set of jobs[1] we have made explicit potential consequences of this occupational segregation on socio-demographic gaps in average earnings. This gap - favorable to the agents who "...t in" the workplace identity - may be a consequence of the fact that, depending on certain conditions, the share of selective jobs is an increasing function of the wage standard under consideration: hiring is more likely to be selective among better-paid jobs. This results from the fact that, all other things equal, a job compensation increases with its degree of demands and that the more demanding a job, the stronger employers' propensity to mobilize intrinsic motivation from workers i.e. to arouse the workplace identity. That is precisely on criteria predisposing certain working persons to the workplace identity that selective hiring occurs in our analysis. Our explanation of socio-demographic gaps in average earnings (as a statistical fact) is thus very simple: female workers and black workers would earn less than white male workers because of their concentration among less demanding jobs.

But still, the assumptions upon which we have developed this argument are restrictive (so that we have talked of "potential consequences") and we have provided little discussion of the conditions of its validity. This new chapter, thus, aims at clarifying the channel through which the kind of occupational segregation we have considered impact on the average earnings of each socio-demographic group within an appropriate framework. In the following, we leave aside the issue of asymmetrical information characterizing the employment relation to study the connection of the trade-o¤ between workplace and out-of-the-workplace identity to market mechanisms (e¤ort is now observable). This choice allows us to more directly explore the conditions for our argument to be valid.

The line followed in this thesis is that non-wage di¤erences between various jobs, on the one hand, the heterogeneity of workers' preferences, on the other hand, justify that employers be endowed with a special market power. As a consequence, in this chapter, we keep assuming that oligopsony prevails in the labor market. Such an assumption allow

---

[1] Jobs are considered as pairs (ϕ, ψ), that is (non-wage grati...cation opportunities, degree of demands).

us to reproduce and clarify the case considered in chapter 4 and to study the impact of competition between employment o¤ers on our argument regarding socio-demographic gaps in average earnings.

The model of this chapter allow us to stress the very nature of our argument: that is a macroeconomic argument. The gap in average earning favorable to the workers of the "dominant" group does not trivially derive from their predisposition to the workplace identity. This predisposition may actually lead them to accept "low" pay relatively to the degree of demands of the job under consideration, such pay level that a worker of the "dominated" group would not have accepted. By providing an example for which the gap in average earning is favorable to the dominated group, we put forward the role of the pool of jobs composition in socio-demographic disparities to emerge.

This chapter includes two parts. The ...rst is devoted to the introduction of our model. We brie‡y situate is among labor market models mobilizing workers preferences on employment conditions: we show that our approach exhibits a double di¤erentiation, both vertical and horizontal. We then introduce a model of labor market with workers manifesting self-esteem motivations. The second part explores, through a simple numerical example, the conditions for our argument to be valid as well as the mechanisms ruling the average earning of various social groups when competition exists between employers in the labor market. In this second step, we particularly study the impact of labor market rationing on welfare.

5.1   Employment conditions and strategic interaction in the labor market

This section is devoted to the introduction of a model of oligopsony in the labor market

5.1.1 Workers' preferences on employment conditions and labor market functioning

The role of workers' preferences on employment conditions are mainly apprehended by the theory of compensating di¤erentials. The idea is that, all other things equals, higher wages compensate less satisfying employment conditions. Perfectly competitive labor market models predictions - in particular the law of single wage - should be understood taking into account employment conditions heterogeneity. Competition between …rms, on one side, between workers, on the other, level out utilities attached to various jobs. This theory lies on objective di¤erences between jobs: di¤erentiation is vertical. In other words, workers' preferences on jobs characteristics are similar.

But workers' preferences on jobs characteristics can also be heterogeneous. A given job may provide di¤erent utility levels depending on the worker under consideration. In such a case, there is horizontal di¤erentiation of jobs. This a¤ects labor market functioning since labor supply put to a particular …rm is then de…ned (contrary to what prevails under perfect competition). Oligopsony models of labor market specify and illustrate the way heterogeneous preferences on labor conditions may a¤ect labor market functioning.

In the model of the employment relation provided earlier (chapter 3), the form of workers' preferences is endogenous: it is de…ned in equilibrium, adjusting in particular to the employment conditions o¤ered by the employer. We have identify employment relations with a pair (ϕ, ψ) where ψ is the degree of demands of the job while the component ϕ is a measure of the non-wage grati…cation opportunities it provides. The important point is that this second component only enters as an argument of the utility function of a worker if he holds the workplace identity. Hence, for a similar degree of demands, two agents, the one holding the out-of-the-workplace identity, the other the workplace identity, may attach the corresponding job di¤erent levels of utility. This justi…es, in the spirit of compensating di¤erential, di¤erent compensation. There is thus heterogeneous employment conditions as well as heterogeneous workers.

For a given type (and/or identity), jobs are vertically di¤erentiated. Some jobs are demanding, others are less: earning gaps re‡ect "objective" di¤erences: vertical differentiation lies on ψ. Between workers of di¤erent types, when equilibrium identity

di¤er, jobs are horizontally di¤erentiated. Workers holding the identity A are sensitive to non-wage grati...cation opportunities provided by their job, others are not: horizontal di¤erentiation lies on the hedonic valuation, or not, of variable ϕ. Employers take this heterogeneity into account.

5.1.2 A model of strategic interaction in the labor market

In the coming model, the issue of asymmetrical information raised in previous chapters remains aside: we focus on market mechanisms.

We take again the model of chapters 3 and 4 with little change, considering agents who choose to achieve self-esteem either through work (identity A) or through other activities outside one's working life (identity B). This choice conditions the form of their utility function. E¤ort is perfectly observed ex post by employers so that a contract only stipulates a transfer[2] $w \geq 0$ when the agent exerts e¤ort (otherwise, employer can punish the agent). We denote J the set of available jobs. Jobs are indexed by $j \in J$, job j being characterized by $\left(S_j, \phi_j, \psi_j \right) \in \mathbb{R}^3_+$. The pro...t of employer j is simply $S_j - w_j$. Let $n_\theta > 0$ be the number of agents with trait $\theta \in \{0, 1\}$. When working in the job j paid w_j, an agent with trait θ's utility writes

$$U_c(j, w_j; \theta) = \begin{cases} (1 + \gamma_w) w_j - \psi_j + I_A \left(\phi_j; \theta \right) & \text{if } c = A \\ w_j - \psi_j + I_B & \text{if } c = B \end{cases}$$

where $I_A \left(\phi_j; \theta \right) = \phi_j - \gamma_w w_A - \gamma_\theta (1 - \theta)$, $\theta \in \{0, 1\}$.

Information and timing. The timing of the market game is as follows. 1) The composition of the pool of labor suppliers (n_0 and n_1) is observed by potential employers. 2) Each ...rm j makes a single hiring o¤er, that is, makes a take-it or leave-it wage o¤er.[3] 3) A ...rst employment applicant is randomly drawn from the pool of labor suppliers, he perfectly observes all hiring o¤ers and choose whether to remain outsider or to become

---

[2] The assumption that transfers must remain positive amounts to assuming that jobs degree of demand ψ is large enough.

[3] An hiring o¤er is not allowed to be contingent upon non-productive traits (such as once gender).

the employee of a ...rm j; in that case, job j is subtracted from the set of hiring o¤ers. 4) A second employment applicant is randomly drawn among remaining labor suppliers; it is his turn to choose whether to accept one of the remaining o¤ers or to be an outsider. 5) The process go one until: all the jobs are ...lled and/or all the applicants have been drawn.

Behaviors, employment conditions and utility. Firms o¤ering a job have two ordered concerns: ...rst, to ...ll the job and make a positive pro...t on it; second, when it is ...lled, to minimize the required transfer i.e. to hire the agent who will do the job for the least pay. By choosing to leave the job un...lled, she guarantees a null pro...t. Hence, job j is ...lled if and only if $w_j \cdot S_j$ and we can restrict employer j's strategy set to $[0, S_j]$.[4]

For all $j \in J$ and $\theta \in \{0, 1\}$ let

$$U_j(w_j; \theta) := \max^{©} (1 + \gamma_w) w_j \text{ } \psi_j + I_A^{\text{ }} \phi_j; \theta^{¢}; w_j \text{ } \psi_j + I_B^{\text{ a}}$$

this function represents the utility of an agent of type θ taken into account his capacity to adjust his identity to o¤ered employment conditions. The fact that this function be indexed by j should not lead to confusions: j refers to o¤ered employment conditions $^{\text{i}} \psi_j, \phi_j^{¢}$. The form of agents' preferences being de...ned but for the trait θ, we are mainly interested in the utility level given employment conditions can arouse for a given pay, which we consider to the pair (j, w_j). By staying at home, an agent gets a utility $I_B > 0$ so that an agent with trait θ accepts the hiring o¤er (j, w_j) if and only if $U_j(w_j; \theta) \geq I_B$.
Let $w_j(U; \theta) = \min \left\{ \frac{\psi_j + U_i \cdot I_A(\phi_j; \theta)}{1 + \gamma_w}; \psi_j + U_i I_B \right\}$: for all $\theta \in \{0, 1\}$, $w_j(.; \theta)$ denotes the inverse function of $U_j(.; \theta)$.

Note that, by putting aside the issue of e¤ort observability, we abolish the di¤erence between strongly and weakly ful...lling jobs. Our typology is restricted to two classes: unful...lling and ful...lling jobs. Let us indicate within this new context conditions of selective hiring (upon socio-demographic criteria). A job is selective if it ful...lling for agents with trait $\theta = 1$. It is non-selective if unful...lling to for agents with trait $\theta = 1$ (a

---

[4] Any strategy $w_j > S_j$ is strictly dominated.

fortiori, it is then unful...lling to agents with trait $\theta = 0$).

We provide in the appendix of the current chapter a general de...nition of labor market equilibrium and prove its existence. Yet, up to now, we cannot provide general results. Failing that, we examine, in the next step, the main mechanisms and implications of our model as regards socio-demographic disparities in earnings on the basis of simple example.

5.2   Market mechanisms and sociodemographic disparities

The purpose of this section is to explore from a simple numerical example, our model mechanism and to draw some consequences as regards socio-demographic disparities.

5.2.1   The elements of the example

Let us present the speci...c assumptions of our example.

Three jobs with distinct characteristics

Let us assume $\gamma_w = \gamma_\theta = \frac{1}{2}$ and $w_A = 1$, and consider three jobs $J = f1, 2, 3g$. These jobs are characterized by

ψnϕ	1	2
1	Job 1	Job 3
2	Job 2	

Job 1 is little demanding but does not provide many non-wage grati...cation opportunities. Job 2 is demanding without providing better non-wage grati...cation opportunities than the job 1. By contrast, the job 3 is both little demanding and source of many non-wage grati...cation opportunities. Job 2 will usually obviously give rise to higher compensations than the two others (compensating di¤erential). For this very reason, the

assignment of this job to an agent with trait 0 or 1 will play a crucial role in resulting socio-demographic gap in average earnings.

All through this illustration, we make the following hypothesis.

Hypothesis It is assumed that for all $j \in \{1, 2, 3\}$, $S_j \geq w_j(I_B; 0)$.

This latter hypothesis guarantees that each ...rm can make a wage o¤er meeting the participation constraint of an agent with trait 0 - a fortiori that of an agent with trait 1 since $w_j(I_B; 0) \geq w_j(I_B; 1)$. This hypothesis is particularly important when agents with trait 1 are scarce.

Previous values imply

$$w_1(U; \theta) = \min \left\{ \frac{2}{3}U + \frac{1}{3}(2 - \theta) ; 1 + U - I_B \right\}$$

$$w_2(U; \theta) = \min \left\{ \frac{2}{3}(U + 1) + \frac{1}{3}(2 - \theta) ; 2 + U - I_B \right\}$$

$$w_3(U; \theta) = \min \left\{ \frac{2}{3}(U - 1) + \frac{1}{3}(2 - \theta) ; 1 + U - I_B \right\}$$

As in the previous chapter, each employer is either indi¤erent between types 0 and 1 workers or strictly prefer type 1 workers. In that case, as in chapter 4, corresponding will be said selective.

Expected average earnings per socio-demographic groups

The average earning of a given socio-demographic group depends on its distribution between di¤erent jobs. This distribution depends itself on the relative frequency of types 0 and 1. The algorithm we have chosen to account for the impact of this relative frequency (on the distribution of socio-demographic groups between the jobs) generally implies that the order in which types 0 and 1 workers apply matter. We want to neutralize that. To do it, we propose to consider expected average earnings given the relative frequency of the types 0 and 1. The point is about weighting average earnings resulting from each con...guration by the probability of this con...guration occurrence.

Let $p(\theta_1, \theta_2, \theta_3)$ denote the probability that the con...guration "job 1 is ...lled by an agent with trait θ_1, job 2 by an agent with trait θ_2, and job 3 by an agent with trait

θ_3", θ_1, θ_2 and θ_3 2 f0, 1g, and $\hat{w}^\theta (\theta_1, \theta_2, \theta_3)$ the corresponding average wage within the population of workers with trait θ. When no agent of one of on group is employed, $\theta_1 = \theta_2 = \theta_3$, we ...x the average wage for this group to 0 - the reservation wage. Hence, we can always compute the expected average wage $\hat{w}^\theta$ for workers with trait θ. It is given by

$$\hat{w}^\theta = \sum_{(\theta_1, \theta_2, \theta_3) \, 2 \, \text{f0,1g}^3} p\left(\theta_1, \theta_2, \theta_3\right) \hat{w}^\theta \left(\theta_1, \theta_2, \theta_3\right)$$

Probability $p\left(\theta_1, \theta_2, \theta_3\right)$ derives from a random successive draw of agents among the work force. It is assumed that, in case of indi¤erence, between available jobs from a drawn agent, the latter chooses each of them with identical probability.

The situation of reference and the steps of the analysis

We successively consider three cases. The ...rst one corresponds to the absence of a signi...cant competition between employers: this is our reference situation. Within a framework where labor supply and demand are both heterogeneous, the case of a simple juxtaposition of three monopsons allows us to focus on the choice, for each job, of a particular type of applicant independently from the choice of other employers. This situation is precisely the one we consider in chapter 4. The second case corresponds to the shortage of type 1 agents (agent with trait $\theta = 1$). In addition to the impact of this shortage on the gap in average pay of groups 0 and 1, we examine consequences in terms of e¢ciency. To this extent, it o¤ers an echo to chapter 3 where we have underlined the impact of the intrinsic motivation on the e¢ciency of the employment relation. The third case deals with a global shortage of agents (i.e. of both types 0 and 1). Which group bene...ts the most of the competition between employers? The point is about echoing to the crowding hypothesis mentioned in the end of the chapter 4.

5.2.2 No signi...cant competition between employers (in the labor market)

Assume that the number of applicants of each type, n_0 and n_1, be large enough so that labor demand is immune from signi...cant competition. For n_θ ¸ 3 whatever θ, ...rms can

a priori behave as monopsons and bind applicants' participation constraint. Whether they be of type 0 or 1, an employee's utility is then I_B and wages

$$w_1(I_B; \theta) = \min\left(\frac{2}{3}I_B + \frac{1}{3}(2 - \theta); 1\right)$$

$$w_2(I_B; \theta) = \min\left(\frac{2}{3}(I_B + 1) + \frac{1}{3}(2 - \theta); 2\right)$$

$$w_3(I_B; \theta) = \min\left(\frac{2}{3}(I_B - 1) + \frac{1}{3}(2 - \theta); 1\right)$$

Job j will be selective if and only if $w_j(I_B; 1) < w_j(I_B; 0)$. Under our initial hypothesis, the selective feature of a job only depends on parameter I_B. We have

Selective jobs as a function of I_B

	$I_B < 1$	$1 \le I_B < 3/2$	$3/2 \le I_B < 2$	$2 \le I_B$
Job 1	selective	non-selective	non-selective	non-selective
Job 2	selective	selective	non-selective	non-selective
Job 3	selective	selective	selective	non-selective

Hence, for $I_B \ge 2$, no employer has a strict preference for type 1 applicants. For given employment conditions, a rise in I_B increases the relative attraction of outside-work, in particular for agents with trait 1. The two types tend to adopt a similar out-of-the-workplace identity which makes them perfect substitutes. By contrast, for $I_B < 1$, outside-work is little attractive, in particular for applicants of type 0. Yet, when the point is about arousing an intrinsic motivation, type 1 applicants have an advantage, jobs are selective.

In the following, we mainly pay attention to the I_B values which make job 2 switch from non-selective to selective. Our point is indeed about illustrating the core argument of chapter 4: all other things equal (particularly non-wage gratification opportunities), the most demanding jobs switch to the set of selective jobs before others. But these jobs are precisely those which require the highest pay. In our example, in the absence of a significant competition between employers, the gap in average pay favorable to type 1 applicants derives from the proportion of fulfilling jobs (to type 1 workers) in the economy. When fulfilling jobs are the minority, agents of type 0 benefit of an expected average pay strictly higher than that of agents with trait 1.

A majority of unful...lling jobs

Let us assume $I_B = \frac{3}{2}$. In that case, in the absence of a signi...cant competition between employers, only the job 3 is selective in the equilibrium (...rm 3 is the only one who favor applicants of type 1). In the equilibrium, jobs 1 and 2 are unful...lling whatever the type of their holder. Job 3 is ful...lling for agents of type 1. Let us denote $\mathbf{w} = (w_1; w_2; w_3)$.

Equilibrium 1 For $I_B = \frac{3}{2}$, $n_1 \geq 1$ and $n_0 + n_1 \geq 3$, employers do not signi...cantly compete in the labor market - they bind their employee's participation constraint. Labor market equilibrium is given by

$$\mathbf{w} = \left(1; 2; \frac{2}{3} + \varepsilon\right)$$

where $\varepsilon = 0$ if $n_1 \geq 3$, $\varepsilon > 0$, $\varepsilon ! 0$ otherwise. The hiring o¤er of ...rm 3 can only be chosen by an agent of type 1 (it violates the participation constraint of type 0 agents).

This equilibrium characterization is provided in the appendix.

As we have seen in chapter 3, ...lling an unful...lling job requires a compete compensation of corresponding demands; by contrast, the ful...lling job is paid below its "objective" disutility.

Let us compare average pay between socio-demographic groups. The strict preference of the employer 3 for type 1 workers entails, for all $\theta_1, \theta_2 \, 2 \, f0, 1g$, $p(\theta_1, \theta_2, 0) = 0$. For $n_1 \geq 3$, an employer targeting a type 1 agent has not to worry that her hiring o¤er meet no demand. For $n_1 = n_0 = 3$ we obtain[5] $\hat{w}^0 = 1,135 > \hat{w}^1 ' 0,961$. Other values of (n_0, n_1) are considered in the appendix.[6] One can in particular consider the case of a total absence of type 0 agents. This absence turns out to be perfectly painless to ...rms: type 1 agent can indeed be substituted. There exists, as we have seen, an asymmetrical substitutability between types. If certain jobs require type 1 workers, such workers are always perfect substitute of type 0 workers. For $1 \cdot n_1 < 3$, at least one job is ...lled with one agent of each type. Firm 3 o¤ers $\frac{2}{3} + \varepsilon$ so that a type 1 agent always favor this o¤er. For $n_1 = n_0 = 2$ one obtains[7] $\hat{w}^0 = \frac{3}{2} > 1,135 > 0,961 > \hat{w}^1 = \frac{21}{24} + \frac{3}{4}\varepsilon$. The gap

---

[5]See the proof in the appendix.
[6]In this ...rst step however, we favor the case of an equal representation of both types applicants in the labor market.
[7]See the proof in the appendix.

in average earnings favorable to type 0 agents is widen. The relative shortage of type 1 agents leads to a reduction in their average pay. This simply results from the fact that the probability that an agent 1 holds jobs 1 or 2 - the better paid - is reduced while job 3 is held by a type 1 agent with probability 1. Contrary to usual intuition, here, the growing shortage of type 1 applicants ampli...es an expected gap in average earnings favorable to type 0 workers.

As long as the most demanding jobs remain unful...lling to agents of type 1, the average pay of agents of type 0 can be higher than that of type 1 agents: the predisposition of type 1 agent for the workplace identity plays negatively on (expected) average earning within group 1. Indeed, employer 3 relies on this predisposition to charge lower pay. Group 1, however, enjoy a "guaranteed" access to employment while group 0 does not - at least so far the number of 1 is large enough. Besides, in the case $n_1 < 3$, type 1 workers enjoy a utility level strictly higher than that of type 0 workers.

The main conclusion of this step (expected average earnings gap favorable to the group 0) is called into question when the more demanding job becomes ful...lling.

A majority of ful...lling jobs to a type 1 worker

Let us consider the case $I_B = \frac{3}{2} ¡ \epsilon$ where $\epsilon > 0$, $\epsilon !$ 0. In this case, employers 2 and 3 strictly prefer agents of type 1.

Equilibrium 1′ For $I_B = \frac{3}{2} ¡ \epsilon$, $n_1 , 2$ and $n_0 + n_1 , 3$, employers do not signi...cantly compete - they bind their employee's participation constraint. Labor market equilibrium is given by

$$\mathbf{w} = \left(1; 2 ¡ \frac{2}{3}\epsilon + \varepsilon; \frac{2}{3} ¡ \frac{2}{3}\epsilon + \varepsilon \right)$$

where $\varepsilon = 0$ if $n_1 , 3$, $\varepsilon > 0$, $\varepsilon !$ 0 otherwise. Hiring o¤ers of ...rms 2 and 3 can only be chosen by type 1 applicants (they violate the participation constraint of type 0 agents).

This equilibrium characterization is provided in appendix.

This marginal reduction in agents' reservation utility does not identically a¤ect the surplus of each ...rm. All other things equal, only ...rms 2 and 3 bene...t from this reduction.

Let us compare expected average earning of each group. For $n_0 = n_1 = 3$, we obtain[8] $\hat{w}^0 = \frac{3}{4} < \hat{w}^1 = \frac{47}{36} \; \text{¡} \; \frac{11}{18}\epsilon$. The expected average earnings gap becomes favorable to type 1 agents. This remains true as $n_0 = n_1 = 2$: $\hat{w}^0 = 1 < \hat{w}^1 = \frac{4}{3} + \frac{4}{3}\varepsilon \; \text{¡} \; \frac{2}{3}\epsilon$.

The fact that the job 2 (the better paid) switch in the set of selective jobs before the job 1 (the more poorly paid) is obviously not accidental. This results from the fact that job 2 is the most demanding (all other things equal). Indeed, the more demanding a job the better paid and the more likely arousing the workplace identity will be pro...table.

Here is illustrated the very idea of the explanation suggested in chapter 4 of the socio-demographic gaps in average earnings: these gaps result from the overrepresentation of the most demanding jobs among selective ones.[9] Indeed, an employer appeal to an intrinsic motivation is, for given non-wage grati...cation opportunities ϕ, all the more likely that the job under consideration is demanding. A job compensation being an increasing function of its degree of demands, its ranking among ful...lling jobs is, all other things equal, all the more likely that it is well-paid and hence an earning gap favorable to agents of type 1.

From now on, we would like to evaluate the impact of a readjustment of market powers (on the labor market) on previous argument. How does this readjustment (favorable to labor supply) a¤ect socio-demographic gaps in average earnings? We have seen below that the relative shortage of type 0 agents (for a total number of applicants higher than 3) was painless to ...rms. We could have also underline that this shortage has no consequences regarding e¢ciency. In the remaining, we successively consider: the case of a shortage of agents of type 1; the case of a global shortage of agents.

5.2.3   Competition between ...rms hiring o¤ers

The main issue raised in the following is about the e¤ects of a shortage of labor supply on the socio-demographic gap in average earnings. Does this shortage increase or reduce the gap between average earnings?

---

[8] See the proof in the appendix.

[9] Ful...lling to type 1 agents.

We distinguish between two kinds of situations. The ...rst assumes an asymmetrical shortage: type 1 applicants are scarce but not those of type 0. Employers targeting type 1 agents in the absence of any shortage have to decide whether to maintain this option (which supposes to increase their wage o¤er) or to make do with type 0 employees. In that case, they must decide whether arousing an intrinsic motivation. We examine the impact of type 1 agents shortage on welfare. The second situation assumes a shortage of both types agents. In a case of equal shortage, how is the gap in average earnings between groups changed? The examination of this question is the occasion to consider the impact of labor return on the gap in average earnings of groups 0 and 1.

A shortage of agents of type 1

We continue the analysis in the going about a new step. The shortage does not concern type 0 agents, we thus assume $n_0 \gtrsim 2$. We show below that the shortage of type 1 agents is favorable, in terms of average earning, to the group 0. The global well-being yet is a¤ected. We consider two con...gurations: in the one, the renunciation to hire type 1 worker goes with a renouncement to stimulate an intrinsic motivation; in the other, ...rm under consideration makes the most of the capacity of type 0 agents to develop an intrinsic motivation.

Type 0 workers are con...ned to unful...lling jobs. As in equilibrium 1', let us assume $I_B = \frac{3}{2} \text{¡} \epsilon, \epsilon > 0, \epsilon \text{!} 0$. Here, the shortage of type 1 agents involves $n_1 = 1$.

Equilibrium 2 For $I_B = \frac{3}{2} \text{¡} \epsilon, \epsilon > 0, \epsilon \text{!} 0$, $n_1 = 1$ and $n_0 \gtrsim 2$, employers 2 and 3 compete to hire a type 1 agent. Employer 3 take the advantage so that 1 and 2 both hire a type 0 applicant. Labor market equilibrium is thus given by

$$\mathbf{w} = \left(1; 2; \frac{2}{3} + \frac{2}{3}\varepsilon \right) \text{ where } \varepsilon > 0, \varepsilon \text{ ! } 0$$

The hiring o¤er of ...rm 3 can only be chosen by an agent of type 1.
This equilibrium is characterized in the appendix.

Job 2 is ful...lling[10] to an agent of type 1... not to an agent of type 0. The con...guration $(0, 0, 1)$ prevails with probability 1, so that $\hat{w}^0 = \frac{3}{2} > \hat{w}^1 = \frac{2}{3}(1 + \varepsilon)$. In terms of earnings, the shortage of agents of type 1 is then favorable to the group 0. The shortage of type 1 applicants, makes job 2 (the better paid) available to type 0 agents.

But still, the shortage of type 1 applicants is globally prejudicial: it leads to a loss in e¢ciency.[11] This situation is an echo of our discussion of chapter 3 on the gains in e¢ciency due to the workplace identity. Grafting the argument of the chapter 4 on this discussion leads us to the welfare consequences of a shortage of type 1 agents: a "free" labor (the work done by intrinsically motivated workers) is subtracted from the system.

Ful...lling jobs are shared. We have mentioned, in chapter 4 conclusion, the advantages of a policy focusing on the development of ful...lling jobs from the perspective of "dominated" groups working persons. Next situation illustrate these advantages in terms of e¢ciency.

Equilibrium 2′ For $I_B = 1 ¡ \epsilon$, $\epsilon > 0$, $\epsilon ! 0$, $n_1 = 1$ and $n_0 , 3$, employers 1, 2 and 3 compete to hire a type 1 worker. The latter is indi¤erent between ...rms 2 and 3 hiring o¤ers but strictly prefer them to that of ...rm 1. Labor market equilibrium leads to

$$\mathbf{w} = \left(1; 2 ¡ \frac{2}{3}\epsilon; \frac{2}{3} ¡ \frac{2}{3}\epsilon \right)$$

Hiring o¤ers of these three ...rms could each be chosen by a type 0 agent.

This equilibrium is characterized in the appendix.

For[12] $n_0 = 3$ and $n_1 = 1$, expected average pays are given by[13] $\hat{w}^0 = \frac{32}{27} ¡ \frac{10}{27}\epsilon > \hat{w}^1 = \frac{8}{9} ¡ \frac{4}{9}\epsilon$. When type 1 agents are scarce, ful...lling jobs (here jobs 2 or 3) become available to type 0 applicants. The capacity of type 0 agents to develop an intrinsic work motivation is used: this moderates the loss in e¢ciency highlighted in the previous case.

---

[10] See equilibrium 1′.

[11] Our reference is the equilibrium 1′. Let us denote $\Omega_{1'}$ the collective well-being - obtained by summing the utilities of employees and the surplus of ...rms - in equilibrium 1′ and Ω_2 collective well-being in equilibrium 2. We have $\Omega_{1'}^{n_1 = 2} > \Omega_2$ and $\Omega_{1'}^{n_1 , 3} > \Omega_2$ for $\epsilon > \varepsilon$.

[12] Here, the assumption that competition is centered on type 1 applicants requires $n_0 , 3$.

[13] See the derivation in appendix.

This case is characterized by a gap between the well-being of type 1 workers and type 0 workers. In jobs 2 and 3, worker 1 bene...ts from the pay which would have been o¤ered to a 0 although these jobs are intrinsically ful...lling: $U_1 = \frac{3}{2} \text{¡} \epsilon >> U_0 (= 1 \text{¡} \epsilon)$. It the shortage of type 1 agents may look unfavorable in terms of pay, this latter situation highlights the bene...ts they draw from is in terms of well-being.

One should note that previous con...guration did not mobilized S_j values beyond our initial hypothesis - $S_j \cdot w_j (I_B; 0)$ for all j. This is due to the fact that hiring a type 0 agent is an option always available to ...rms. How does employers' exposure to an intensi...ed competition a¤ect the gap in average earnings between groups?

The case of a global shortage of the labor force.

We content ourselves with the case $n_0 = n_1 = 1$ and assume, as in equilibrium 1, $I_B = \frac{3}{2}$. The role of the gross surpluses S_j becomes critical. Indeed, one of the three ...rms is then excluded from labor market, the less o¤ering.

Numerical assumptions. The initial hypothesis involves: $S_1 \, \text{¸} \, 1$, $S_2 \, \text{¸} \, 2$ and $S_3 \, \text{¸} \, 1$. The fact that job 3 o¤ers large non-wage grati...cation opportunities compared to the two others endows employer 3 with special market power. In this section, we would like to explore the e¤ect of a gradual readjustment of the balance regarding market power (in favor of employers 1 and 2) on the average earnings gap between groups. To do that, we maintain the gross surplus of job 3 constant while considering a parallel increase of the gross surplus of jobs 1 and 2. Our choice as regards numerical assumptions, thus, aims at: i) illustrating the evolution of the gap between earnings as ...rms 1 and 2 intensify the competitive pressure on ...rm 3; ii) illustrating the reversal of the earnings gap as a consequence of the crowding out of employer 3. Yet, in this choice, we have cared about maintaining a common scale for the surpluses of ...rms 2 and 3: the gross surplus of ...rm 2 per unity of demands gradually increases up to exceeding job 3 surplus. Moreover, most of the con...guration likely to occur (see the appendix) are considered.

What is at stake here is simply having clearer ideas of the mechanisms likely to play in the case of a global shortage of labor. We consider the six following cases.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
S_1	$\frac{28}{20}$	$\frac{31}{20}$	$\frac{33}{20}$	$\frac{34}{20}$	$\frac{36}{20}$	$\frac{44}{20}$
$S_2{}^i = \frac{3}{2}S_1{}^\mathrm{c}$	$\frac{84}{40}$	$\frac{93}{40}$	$\frac{99}{40}$	$\frac{102}{40}$	$\frac{108}{40}$	$\frac{132}{40}$
S_3	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{4}{3}$

That is, in terms of gross surplus per unity of demands:

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
$\frac{S_1}{\psi_1}$ '	1, 40	1, 55	1, 65	1, 70	1, 80	2, 2
$\frac{S_2}{\psi_2}$ '	1, 05	1, 16	1, 23	1, 27	1, 35	1, 65
$\frac{S_3}{\psi_3}$ '	1, 33	1, 33	1, 33	1, 33	1, 33	1, 33

Results and discussion. The reference situation (Ref.) is the equilibrium 1 where employers do not compete. This is the relevant reference to the extent that gross surplus do not have a part beyond the initial hypothesis: whatever the case under consideration (1 to 6), the single di¤erence to equilibrium 1 actually lies on applicants shortage. Results of the analysis are presented in the next table.

Impact of the competition between ...rms hiring o¤ers $I_B = \frac{3}{2}$¢

	$n_0 + n_1 \geq 3$ $n_1 \geq 1$	$n_0 = n_1 = 1$					
	Ref.	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
w_1	1	$\frac{132}{120}+\varepsilon$	$\frac{159}{120}+\varepsilon$	$\frac{177}{120}+\varepsilon$	$\frac{186}{120}+\varepsilon$		$\frac{200}{120}+\varepsilon$
w_2	2	n. f.[1]	n. f.	n. f.	n. f.		$\frac{320}{120}+\varepsilon$
w_3	$\frac{2}{3}+\varepsilon$	$\frac{92}{120}+\varepsilon$	$\frac{119}{120}+\varepsilon$	$\frac{137}{120}+\varepsilon$	$\frac{146}{120}+\varepsilon$		n. f.
$\frac{w_1 - w_1^{Ref.}}{w_1^{Ref.}}$	0	0, 10	0, 32	0, 47	0, 55		0, 66
$\frac{w_2 - w_2^{Ref.}}{w_2^{Ref.}}$	0	n. d.[2]	n. d.	n. d.	n. d.		0, 33
$\frac{w_3 - w_3^{Ref.}}{w_3^{Ref.}}$	0	0, 15	0, 48	0, 71	0, 82		n. d.
$\hat{w}^0$	$[1; 2]$	$\frac{132}{120}+\varepsilon$	$\frac{159}{120}+\varepsilon$	$\frac{177}{120}+\varepsilon$	$\frac{186}{120}+\varepsilon$		$\frac{200}{120}+\varepsilon$
$\hat{w}^1$	$\left[\frac{80}{120}+\varepsilon; \frac{160}{120}+\frac{1}{2}\varepsilon\right]$	$\frac{92}{120}+\varepsilon$	$\frac{119}{120}+\varepsilon$	$\frac{137}{120}+\varepsilon$	$\frac{146}{120}+\varepsilon$		$\frac{320}{120}+\varepsilon$
$\frac{\hat{w}^1 - \hat{w}^0}{\hat{w}^0}$	-	$-0, 30$	$-0, 25$	$-0, 22$	$-0, 21$		$0, 60$

(1) non-...lled job; (2) non-de...ned.

The job 2 is obviously excluded from the market in the caser 1 to 4: that is the more demanding job. Unless its gross surplus be particularly high, ...rm o¤ering the most demanding jobs are the ...rst to su¤er as a result of a global shortage of applicants.

Let us start by discussing cases 1 to 4. The shortage obviously results in an increase in earnings. Previous results allow to observe that the pace of this increase di¤er depending on whether one considers the job 1 (...lled with a type 0 agent) or 3 (...lled with a type 1 agent). We have computed the relative gap to the reference situation for these jobs. It turns out that (see the appendix) competition is focused on the sole employer 2 exclusion which is, as we said earlier, particularly interested in hiring a type 1 agent. Employer 2, thus, exerts a stronger pressure on the employer 3 than on the employer 1 and hence the asymmetry as regards the pace of the increases in equilibrium wages 1 and
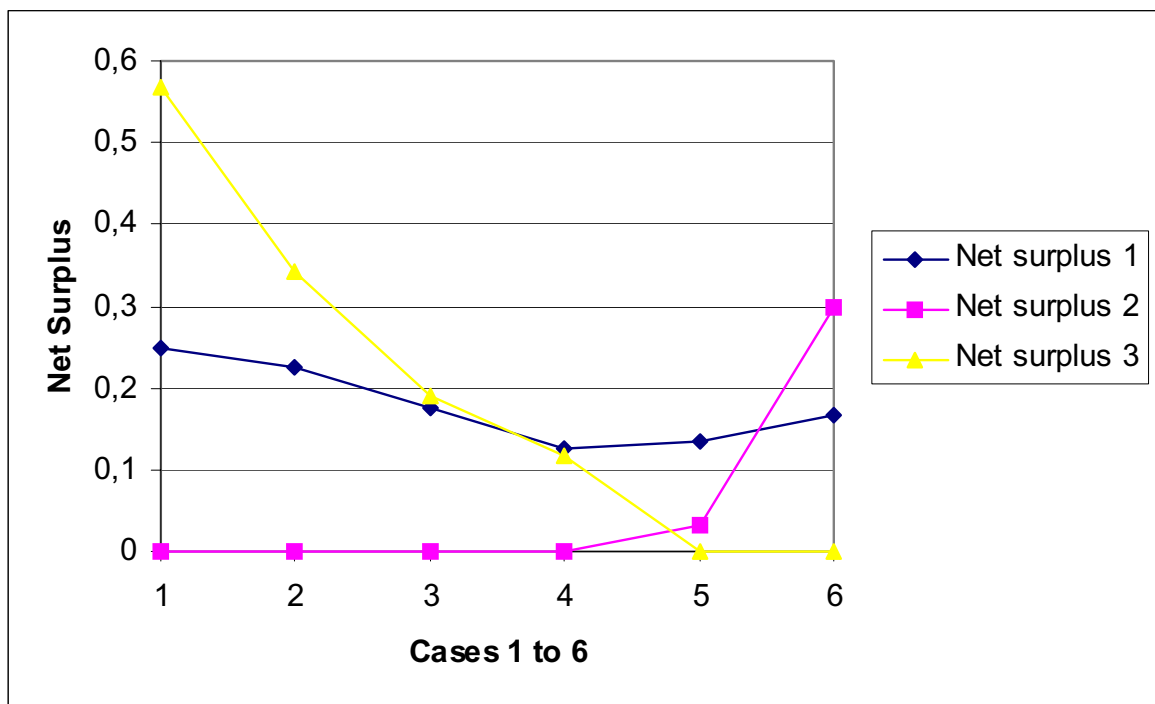
Figure 5-1: Net surplus for jobs 1, 2, and 3.

3. The evolution of the earning gap between the groups - favorable to type 0 workers (see equilibrium 1) - re‡ects previous observation: the increase of labor return in job 2 is relatively more favorable to agents of type 1 than to agent of type 0.

The reversal occurring between cases 4 and 5 underlines the observation we made by distinguishing equilibria 1 and 1′. The earnings gap favorable to the agents of type 1 is mostly due to their prevalence among the most demanding jobs. That is why the crowding out of the ...rm 3 by employer 2 involves an (average) earnings gap favorable to type 1 workers.

Nous proposons avec les ...gures 5-1 et 5-2 l'illustration graphique des principaux aspects de l'analyse.

The important point of this analysis seem the following. For comparable gross surplus, competition is unfavorable to the employers o¤ering the most demanding jobs. Corresponding ...rms, hence, tend to be crowd out from the labor market. Yet, the pressure
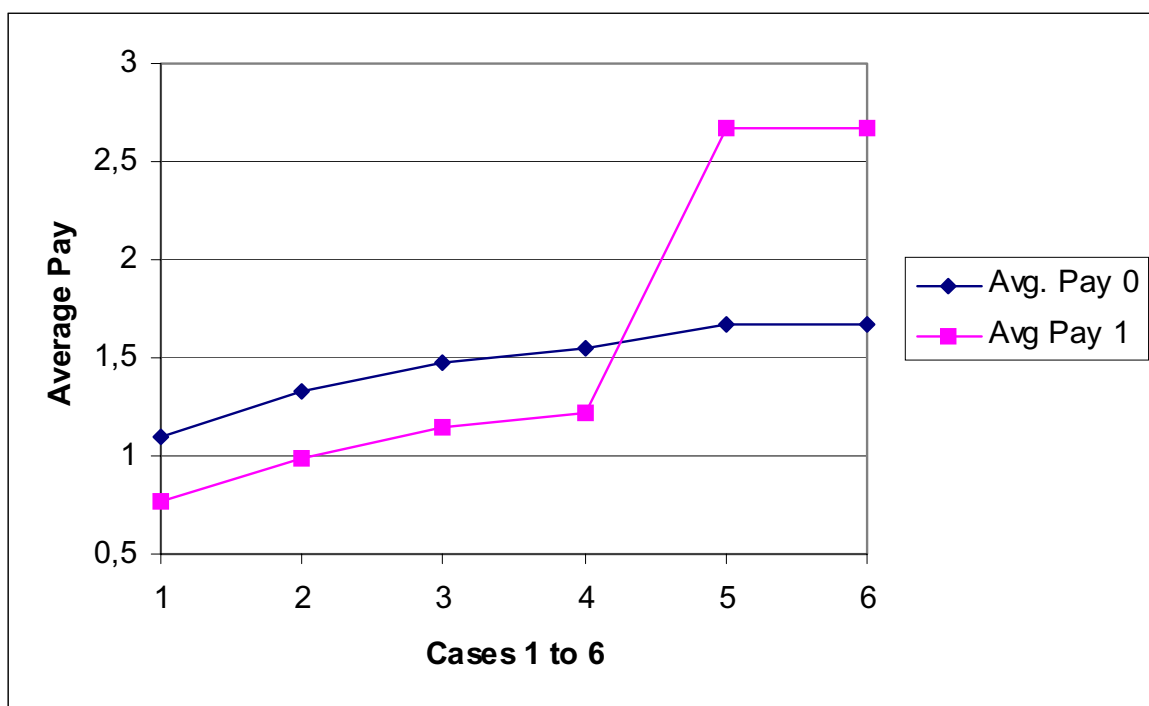
206

Figure 5-2: Average earnings for workers of type 0 and 1.

their existence exerts on operating ...rms is asymmetric. Indeed, ...rms o¤ering the most demanding jobs are also the most prone to solicit type 1 agents. Maintaining outside the market is thus more costly to ...rms employing type 1 workers than to others.

Summary and conclusion

In this chapter, we have considered a series of con...gurations illustrating how our argument of self-esteem achievement through work interplays with labor market functioning.

The case of no signi...cant competition between employers corresponded to the situation examined in chapter 4. We have established that, if the "dominant" socio-demographic group could bene...t from a guaranteed access to employment, nothing in our micro model assumptions predetermine a gap in average earning favorable to this group. The average earnings gap favorable to agents of type 1 does not trivially derives from their predisposition to the workplace identity. Indeed, this predisposition may lead them to accept "low" pays relatively to the degree of demands of the jobs under consideration, pay levels that a 0 agent would not have accepted. In the example considered above, when unful...lling jobs were the majority, group 1 has an expected average pay lower than that of the group 0. This is only when ful...lling jobs are a majority that the earnings gap becomes favorable to the group 1. The example shows that, all other things equal, a job is all the more likely to require the workplace identity (and then to be selective) that it is more demanding and then better paid.

We have also explored the consequences of a shortage of type 1 agents. A ...rst situation (where type 0 agents keep holding unful...lling jobs) allows us to illustrate of the consequences in terms of e¢ciency, of an under-utilization of the capacity of employees to develop an intrinsic work motivation. A second situation illustrated the policy prescription made in chapter 4 conclusion: designing jobs which are ful...lling from the point of view of agents belonging to the "dominated" group. We could have shown that this allows to moderate the loss in e¢ciency resulting from the shortage of type 1 agents.

The issue of a special "shortage" of type 0 agents has been brie‡y raised. In fact, we have highlighted that the asymmetric substitutability of applicants 0 and 1 (agent 1 is always substitutable to an agent 0 but the reverse is false) excluded that a "shortage"

of type 0 workers be detrimental to any employer in the absence of a global shortage of applicants. The last step of our analysis precisely dealt with this case. This led us to see the relative bene...ts drawn by each group from its shortage. The increases in average pay obtained by type 1 workers are always higher than that of type 0 workers. In other words, a global shortage in labor supply seem to be more bene...cial to the workers of the "dominant" group. This results from the fact that competition tends to push the most demanding jobs aside. The competitive pressure exerted by this outsider seems asymmetrical: since corresponding employers are mostly interested by agents of the "dominant" group, they impose their actual employers higher wage increases.

Previous observation remain temporary conclusions. De...nitive results require the analytical characterization of the labor market equilibrium in the general case, which still need to be achieved. The general model provided in appendix allow to make oneself ideas as regards to the obstacle we meet.

A last remark deserves attention which deal with the structure of the market game introduced above. Our assumption as regards the sequence characterizing the match of labor supply and demand may appear as dissatisfying: its at least clearly arbitrary. Yet, we believe the observation we derived from this assumption should not be strongly a¤ected. In our point of view, this assumption has a functional rather than substantial scope: it is rather an algorithm allowing the characterization of an equilibrium matching. We could have used more sophisticated algorithms such as that of Gale and Shapley usually used in matching models. The important point is to obtain stable equilibria which we believe is the case here.

Appendix

In current appendix, one will ...nd the calculation of expected average earnings $\hat{w}^0$ and $\hat{w}^1$ in the di¤erent cases considered above, the characterization of equilibria 1 to 3.6 and an attempt of general model.

5.3 The calculation of expected average earnings of groups 0 and 1

This calculation comprises two steps. The ...rst consists in determining the equilibrium earnings for each employment con...guration; the second step is to calculate their respective probability. The procedure to calculate the probability of the di¤erent con...gurations occurrence derives from the conditions of the match of labor o¤er and demand. It exhibits little variation from one case to the other but deserves attention for qualitative results depend on this calculation. We present though the calculation in the main cases.

5.3.1 No signi...cant competition between employers

We successively consider the cases in which unful...lling jobs are the majority and the minority.

A majority of unful...lling jobs.

Employment con...gurations depend on the number of agents for each type.

The case n_1 ¸ 3. Average pays in each employment con...gurations are given by

	$\hat{w}^\theta(1,1,1)$	$\hat{w}^\theta(1,0,1)$	$\hat{w}^\theta(0,1,1)$	$\hat{w}^\theta(0,0,1)$
$\theta = 0$	0	2	1	$\frac{3}{2}$
$\theta = 1$	$\frac{11}{9}$	$\frac{5}{6}$	$\frac{4}{3}$	$\frac{2}{3}$

The point now is about establishing the probability of the various con...gurations when n_1 ¸ 3. Under this condition, type 1 agents are indi¤erent between the three jobs, type 0 agents, by contrast, are only indi¤erent between the jobs 1 and 2. The hiring o¤er of employer 3 violate their participation constraint.

² The set of draws compatible with $(\theta_1, \theta_2, \theta_3) = (1,1,1)$ comprises: $(1,1,1,...)$ and $(1,1,0,...)$. The ...rst of these draws lead to $(1,1,1)$ with probability 1. The second only

leads to it if neither the ...rst drawn agent nor the second do choose the job 3. This occurs with probability $\frac{1}{3}$. As a consequence:

$$p(1,1,1) = \frac{n_1}{n_0+n_1}\frac{n_1-1}{n_0+n_1-1}\frac{n_1-2}{n_0+n_1-2} + \frac{1}{3}\frac{n_1}{n_0+n_1}\frac{n_1-1}{n_0+n_1-1}\frac{n_0}{n_0+n_1-2}$$

$$p(1,1,1) = \frac{n_1}{n_0+n_1}\frac{n_1-1}{n_0+n_1-1}\frac{n_1-2+\frac{1}{3}n_0}{n_0+n_1-2}$$

[2] The set of draws compatible with the con...guration $(1,0,1)$ are $(1,0,0,...)$, $(0,1,0,...)$, $(0,1,1,...)$, $(1,1,0,...)$ and $(1,0,1,...)$. The ...rst draw leads to con...guration $(1,0,1)$ with probability $\frac{1}{3}$, the second with probability $\frac{1}{4}$, the third with probability $\frac{1}{2}$, the last with probability $\frac{1}{3}$. Hence

$$p(1,0,1) = \frac{1}{3}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2} + \frac{1}{4}\frac{n_0}{n_0+n_1}\frac{n_1}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2}$$
$$+ \frac{1}{2}\frac{n_0}{n_0+n_1}\frac{n_1}{n_0+n_1-1}\frac{n_1-1}{n_0+n_1-2} + \frac{1}{3}\frac{n_1}{n_0+n_1}\frac{n_1-1}{n_0+n_1-1}\frac{n_0}{n_0+n_1-2}$$
$$+ \frac{1}{2}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_1-1}{n_0+n_1-2}$$

$$p(1,0,1) = \frac{7}{12}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2} + \frac{4}{3}\frac{n_1}{n_0+n_1}\frac{n_1-1}{n_0+n_1-1}\frac{n_0}{n_0+n_1-2}$$

[2] The draws compatible with $(0,1,1)$ are $(1,0,0,...)$, $(0,1,0,...)$, $(1,0,1,...)$, $(0,1,1,...)$, $(1,1,0,...)$. The ...rst leads to con...guration $(0,1,1)$ with probability $\frac{1}{3}$, the second with probability $\frac{1}{4}$, the third with probability $\frac{1}{3}+\frac{1}{3}\frac{1}{2}$, the fourth with probability $\frac{1}{2}$, the last with probability $\frac{1}{3}\frac{1}{2}+\frac{1}{3}\frac{1}{2}=\frac{1}{3}$. Hence

$$p(0,1,1) = \frac{1}{3}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2} + \frac{1}{4}\frac{n_0}{n_0+n_1}\frac{n_1}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2}$$
$$+ \frac{1}{2}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_1-1}{n_0+n_1-2} + \frac{1}{2}\frac{n_0}{n_0+n_1}\frac{n_1}{n_0+n_1-1}\frac{n_1-1}{n_0+n_1-2}$$
$$+ \frac{1}{3}\frac{n_1}{n_0+n_1}\frac{n_1-1}{n_0+n_1-1}\frac{n_0}{n_0+n_1-2}$$

$$p(0,1,1) = \frac{7}{12}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2} + \frac{4}{3}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_1-1}{n_0+n_1-2}$$

² The draws compatible with con…guration $(0, 0, 1)$ are $(1, 0, 0, ...)$, $(0, 1, 0, ...)$, $(0, 0, 1, ...)$, $(0, 0, 0, ...)$. The …rst leads to con…guration $(0, 0, 1)$ with probability $\frac{1}{3}$, the second with probability $\frac{1}{2}$, both the third and fourth both with probability 1. Hence

$$p(0,0,1) = \frac{1}{3}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2} + \frac{1}{2}\frac{n_0}{n_0+n_1}\frac{n_1}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2}$$
$$+ \frac{n_0}{n_0+n_1}\frac{n_0-1}{n_0+n_1-1}\frac{n_1}{n_0+n_1-2} + \frac{n_0}{n_0+n_1}\frac{n_0-1}{n_0+n_1-1}\frac{n_0-2}{n_0+n_1-2}$$

$$p(0,0,1) = \frac{11}{6}\frac{n_1}{n_0+n_1}\frac{n_0}{n_0+n_1-1}\frac{n_0-1}{n_0+n_1-2} + \frac{n_0}{n_0+n_1}\frac{n_0-1}{n_0+n_1-1}\frac{n_0-2}{n_0+n_1-2}$$

² One can easily check that

$$\sum_{(\theta_1,\theta_2,\theta_3)\in f0,1g^3} p(\theta_1, \theta_2, \theta_3) = 1$$

The probability of occurrence of the various con…gurations are given by $p(1,1,1) = \frac{n_1(n_1-1)}{D}\left(\frac{1}{3}n_0 + n_1 - 2\right)$, $p(1,0,1) = p(0,1,1) = \frac{n_1 n_0}{D}\left(\frac{7}{12}n_0 + \frac{4}{3}n_1 - \frac{23}{12}\right)$ and $p(0,0,1) = \frac{n_0(n_0-1)}{D}\left(n_0 + \frac{11}{6}n_1 - 2\right)$ where $D = (n_0 + n_1)(n_0 + n_1 - 1)(n_0 + n_1 - 2)$. For $n_0 = n_1 = 3$: $p(1,1,1) = \frac{40}{400}$, $p(1,0,1) = p(0,1,1) = \frac{115}{400}$, $p(0,0,1) = \frac{130}{400}$ and hence $\hat{w}^0 = 1,135 > \hat{w}^1 \simeq 0,961$. Expected average earnings are calculated for some values of n_0 and n_1 in the next table.

$n_0 \cap n_1$	3	4	5
0	$(0; 1, 222)$	$(0; 1, 222)$	$(0; 1, 222)$
1	$(1; 1, 129)$	$(0, 983; 1, 229)$	$(0, 666; 1, 160)$
2	$(1, 25; 1, 030)$	$(1, 1; 1, 069)$	$(0, 976; 1, 347)$
3	$(1, 35; 0, 961)$	$(1, 242; 1, 007)$	$(1, 142; 1, 040)$
4	$(1, 4; 0, 914)$	$(1, 321; 0, 960)$	$(1, 269; 1, 097)$
5	$(1, 428; 0, 879)$	$(1, 369; 0, 924)$	$(1, 305; 1, 093)$

The case $1 \cdot n_1 < 3$. Average pay in each con…guration are given by

$$\hat{w}^\theta(1,0,1) \qquad \hat{w}^\theta(0,1,1) \qquad \hat{w}^\theta(0,0,1)$$

$$\theta = 0 \qquad\quad 2 \qquad\qquad 1 \qquad\qquad \tfrac{3}{2}$$

$$\theta = 1 \qquad \tfrac{5}{6} + \tfrac{1}{2}\varepsilon \qquad \tfrac{4}{3} + \tfrac{1}{2}\varepsilon \qquad \tfrac{2}{3} + \varepsilon$$

The probabilities of various con...gurations when $n_1 < 3$. Con...guration $(1,1,1)$ cannot occurs since, at most, two type 1 applicants are available: its probability is zero.

² The draws compatible with con...guration $(1,0,1)$ are $(1,1,0,...)$, $(1,0,1,...)$, $(0,1,1,...)$. the ...rst leads to con...guration $(1,0,1)$ with probability $\tfrac{1}{2}$, the second with probability $\tfrac{1}{2}$, the third with the probability $\tfrac{1}{2}$. Hence

$$p(1,0,1) = \frac{3}{2} \frac{n_0}{n_0 + n_1} \frac{n_1}{n_0 + n_1 - 1} \frac{n_1 - 1}{n_0 + n_1 - 2}$$

the reasoning is the same as regards con...guration $(0,1,1)$ and on obtains $p(0,1,1) = p(1,0,1)$.

² The draws compatible with $(0,0,1)$ are $(0,0,0,...)$, $(1,0,0,...)$, $(0,1,0,...)$, $(0,0,1,...)$. The ...rst leads to $(0,0,1)$ with probability 1, as well as all other draws. Hence

$$p(0,0,1) = \frac{n_0}{n_0 + n_1} \frac{n_0 - 1}{n_0 + n_1 - 1} \frac{n_0 - 2}{n_0 + n_1 - 2} + 3\frac{n_1}{n_0 + n_1} \frac{n_0}{n_0 + n_1 - 1} \frac{n_0 - 1}{n_0 + n_1 - 2}$$

² The probabilities of these con...gurations to occurs are given by $p(1,0,1) = \tfrac{3}{2}\frac{n_0 n_1 (n_1 - 1)}{D}$, $p(0,1,1) = \tfrac{3}{2}\frac{n_0 n_1 (n_1 - 1)}{D}$ and $p(0,0,1) = (n_0 + 3n_1 - 2)\frac{n_0(n_0 - 1)}{D}$.

For $n_0 = n_1 = 2$: $p(0,0,1) = \tfrac{1}{2}$, $p(1,0,1) = p(0,1,1) = \tfrac{1}{4}$.

The reasoning remains the same for the other cases.

A majority of ful...lling jobs.

Two cases must be distinguished.

For $n_1 \geq 3$. Under this assumption $\varepsilon = 0$ so that average pays take the following values

$$\bar{w}^{\theta}(1,1,1) \quad \bar{w}^{\theta}(0,1,1)$$

$$\theta = 0 \qquad 0 \qquad\qquad 1$$

$$\theta = 1 \qquad \tfrac{11}{9} - \tfrac{4}{9}\epsilon \qquad \tfrac{4}{3} - \tfrac{2}{3}\epsilon$$

For all $\theta_1, \theta_2, \theta_3 \in \{0,1\}$, $p(\theta_1, \theta_2, 0) = p(\theta_1, 0, \theta_3) = p(\theta_1, 0, 0) = 0$. The probabilities of remaining con...gurations are

$$p(1,1,1) \;=\; \frac{n_1(n_1 - 1)(n_1 - 2)}{D} + \frac{2}{3}\frac{n_0 n_1 (n_1 - 1)}{D} + \frac{1}{3}\frac{n_0 n_1}{(n_0 + n_1)(n_0 + n_1 - 1)}$$

$$p(0,1,1) \;=\; \frac{n_0}{n_0 + n_1} + \frac{2}{3}\frac{n_0 n_1}{(n_0 + n_1)(n_0 + n_1 - 1)} + \frac{1}{3}\frac{n_0 n_1 (n_1 - 1)}{D}$$

For $n_0 = n_1 = 3$: $p(1,1,1) = \tfrac{1}{4}$, $p(0,1,1) = \tfrac{3}{4}$.

For $n_1 = 2$. Under this assumption $\varepsilon > 0$ and $p(0,1,1) = 1$. The average pays for each group are

$$\bar{w}^{\theta}(0,1,1)$$

$$\theta = 0 \qquad 1$$

$$\theta = 1 \qquad \tfrac{4}{3} + \tfrac{4}{3}\varepsilon - \tfrac{2}{3}\epsilon$$

The fact that there is only two applicants of type 1 guarantees that an agent of type 0 be employed and hence a reduced earning gap.

5.3.2 A shortage of agents of type 1

Where ful...lling jobs are shared.

Possible con...gurations are $(0,1,0)$, $(0,0,1)$ and, if $n_0 \geq 3$, $(0,0,0)$.

[2] The draws compatible with the con...guration $(0,0,0)$ - if this con...guration is possible - are $(0,0,1,...)$ and $(0,0,0,...)$. The ...rst draw leads to $(0,0,0)$ with probability $\tfrac{1}{3}$, the second with probability 1. One obtains

$$p(0,0,0) = \begin{cases} \tfrac{1}{3}\tfrac{1}{n_0+1} + \tfrac{n_0 - 2}{n_0+1} & \text{if } n_0 \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

214

$(0,0,1,...)$. The …rst draw leads to con…guration $(0,1,0)$ with probability $\frac{1}{2}$, the second with probability $\frac{1}{2}$, the third with probability $\frac{1}{3}$. Hence

$$p(0,1,0) = \frac{4}{3}\frac{1}{n_0 + 1}$$

The reasoning is similar as regards con…guration $(0,0,1)$ and one obtains $p(0,0,1) = p(0,1,0)$.

5.4 Characterization of equilibria in the presence of competitive pressures

We consider successively the case of a shortage of type 1 agents and that of a global shortage of applicants.

5.4.1 Shortage of agents of type 1

Workers of type 0 are led to substitute to type 1 lacking applicants. This can be accompanied are not by the renouncement from the employers to stimulate an intrinsic motivation.

The case $I_B = \frac{3}{2}$ ¡ ϵ, $\epsilon > 0$, $\epsilon !$ 0.

In this case, the shortage of type 1 agents involves $n_1 = 1$. Previous step indicate that …rms 2 and 3 will compete, that is the …rms which would have favored type 1 agents if their reservation utility had been I_B. If she had to renounce hiring a type 1 agent, each employer j 2 f2, 3g would …ll her job with a type 0 agent. This type being abundant, j could limit her wage o¤er to the level binding a 0 participation constraint. Minimal

required pay would then be

$$w_2(I_B;0) = \min\left\{\frac{2}{3}(I_B+1)+\frac{2}{3};2\right\}^{\frac{3}{4}} = 2 > w_2(I_B;1) = 2 - \frac{2}{3}\epsilon$$

$$w_3(I_B;0) = \min\left\{\frac{2}{3}(I_B-1)+\frac{2}{3};1\right\}^{\frac{3}{4}} = 1 - \frac{2}{3}\epsilon > w_3(I_B;1) = \frac{2}{3}-\frac{2}{3}\epsilon$$

Hence, each firm j will raise the bidding so long $w_j \cdot w_j(I_B;0)$. Let $\overline{U}_{1j}$ denote the utility experienced by an agent of type 1 by the highest pay offer employer j might make. Previous discussion allow to state that $\overline{U}_{1j}$ is defined by $w_j\left(\overline{U}_{1j};1\right) = w_j(I_B;0)$. The fact that it is beneficial to firm j to target the type 1 involves the arousing of the workplace identity. As a consequence, $w_2\left(\overline{U}_{12};1\right) = \frac{2}{3}\left(\overline{U}_{12}+1\right)+\frac{1}{3}$ and $w_3\left(\overline{U}_{13};1\right) = \frac{2}{3}\left(\overline{U}_{13}-1\right)+\frac{1}{3}$ and hence[14] $\overline{U}_{12} = \frac{3}{2} < \overline{U}_{13} = 2 - \epsilon$. Employer 3 is assured to prevail. She contents herself with guaranteeing to the agent of type 1 a utility level $U_1 = \overline{U}_{12}+\varepsilon$, $\varepsilon > 0$, $\varepsilon \,!\, 0$, that is $U_1 = \frac{3}{2}+\varepsilon$. The pay offer made by employer 3 is then $w_3(U_1;1) = \frac{2}{3}\left(\frac{1}{2}+\varepsilon\right)+\frac{1}{3} = \frac{2}{3}+\frac{2}{3}\varepsilon$. The fall back position of firm 2 is to hired a type 0 agent for a wage $w_2(I_B;0) = 2$. Workplace identity is not aroused: job 2 is then unfulfilling!

The case $I_B = 1 - \epsilon$, $\epsilon > 0$, $\epsilon \,!\, 0$.

If type 1 applicants were abundant, the three employers would favor them over type 0 agents. With this new value of I_B, one obtains following values

	$w_j(I_B;0)$	$w_j(I_B;1)$	$\overline{U}_{1j} = U_j(w_j(I_B;0);1)$
Emploi 1	1	$1 - \frac{2}{3}\epsilon$	1
Emploi 2	$2 - \frac{2}{3}\epsilon$	$\frac{5}{3}-\frac{2}{3}\epsilon$	$\frac{3}{2}-\epsilon$
Emploi 3	$\frac{2}{3}-\frac{2}{3}\epsilon$	$\frac{1}{3}-\frac{2}{3}\epsilon$	$\frac{3}{2}-\epsilon$

When participation constraint is binding: job 1 is unfulfilling to agents of type 0 but is now fulfilling to agents of type 1; job 2 is fulfilling whatever the type of its holder, as well as job 3. The examination of the utilities brought to an agent of type 1 by the best

---

[14] One can check that for $\overline{U}_{12} = \frac{3}{2}$ and $I_B = \frac{3}{2}-\epsilon$, it is true that $\frac{2}{3}\left(\overline{U}_{12}+1\right)+\frac{1}{3} < 2+\overline{U}_{12}-I_B$.

o¤er of each ...rm shows: ...rstly, that ...rm 1 is pushed aside (for agents 1 hiring); secondly, that ...rms 2 and 3 do not succeed in di¤erentiating themselves. Their competition to attract agent of type 1 leads them to make wage o¤ers to this type of applicant such that they are just indi¤erent between hiring type 0 or type 1 workers.

For $n_0 = 3$ and $n_1 = 1$, $p(0,1,0) = p(0,0,1) = p(0,0,0) = \frac{1}{3}$. Average pay in each con...gurations are given by

	$\hat{w}^\theta(0,1,0)$	$\hat{w}^\theta(0,0,1)$	$\hat{w}^\theta(0,0,0)$
$\theta = 0$	$\frac{5}{6} - \frac{1}{3}\epsilon$	$\frac{3}{2} - \frac{1}{3}\epsilon$	$\frac{11}{9} - \frac{4}{9}\epsilon$
$\theta = 1$	$2 - \frac{2}{3}\epsilon$	$\frac{2}{3} - \frac{2}{3}\epsilon$	0

and hence the expected average earnings we obtain in the chapter's body.

5.4.2 A global shortage of applicants $\left(I_B = \frac{3}{2} \right)$

Competition can be polarized or not. The best wage o¤er each ...rm can make to a type $\theta \in \{0,1\}$ agent is $w_j = S_j$. Corresponding utility levels are

$$U_1(S_1; \theta) = \max \left\{ \frac{3}{2}S_1 + \frac{1}{2}\theta - 1; S_1 + \frac{1}{2} \right\}$$

$$U_2(S_2; \theta) = \max \left\{ \frac{3}{2}S_2 + \frac{1}{2}\theta - 2; S_2 - \frac{1}{2} \right\}$$

$$U_3(S_3; \theta) = \max \left\{ \frac{3}{2}S_3 + \frac{1}{2}\theta; S_3 + \frac{1}{2} \right\}$$

Let U_θ denote the utility obtained by the type θ agent in labor market equilibrium.

$$w_1(U_\theta; \theta) = \min \left\{ \frac{2}{3}U_\theta + \frac{1}{3}(2 - \theta); U_\theta - \frac{1}{2} \right\}$$

$$w_2(U_\theta; \theta) = \min \left\{ \frac{2}{3}(U_\theta + 1) + \frac{1}{3}(2 - \theta); U_\theta + \frac{1}{2} \right\}$$

$$w_3(U_\theta; \theta) = \min \left\{ \frac{2}{3}(U_\theta - 1) + \frac{1}{3}(2 - \theta); U_\theta - \frac{1}{2} \right\}$$

Polarized competition: cases 1, 2, 3 and 4. Following cases correspond to a balance of powers such that competition is polarized on the exclusion of a single ...rm from the labor market.

Case 1   implies $U_3(S_3;\theta) > U_1(S_1;\theta) > U_2(S_2;\theta)$ whatever $\theta \in \{0,1\}$. Job 2 does not exhibit a large enough gross surplus to allow to the corresponding ...rm to make appealing hiring o¤ers. Yet, this ...rm exerts a pressure on the two other ...rms as a potential entrant: its presence raise the utility reservation of each type of agent. For $U_{(0)} = U_2(S_2;0) = \frac{8}{5}$ and $U_{(1)} = U_2(S_2;1) = \frac{33}{20}$, $0 \bar{A}_1 1$, since $w_1\left(U_{(0)};0\right) = \frac{11}{10} < \frac{23}{20} = w_1\left(U_{(1)};1\right)$, and $0 \acute{A}_3 1$ since $w_3\left(U_{(0)};0\right) = \frac{16}{15} > \frac{23}{30} = w_3\left(U_{(1)};1\right)$. Employers 1 and 3 do not directly compete with each other - their plans are mutually compatible. In labor market equilibrium, types 0 and 1 utilities are hence $U_0 = U_{(0)} + \varepsilon = \frac{8}{5} + \varepsilon$ and $U_1 = U_{(1)} + \varepsilon = \frac{33}{20} + \varepsilon$.

Equilibrium 3.1 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = \left(\frac{28}{20}, \frac{84}{40}, \frac{4}{3}\right)$ employer 2 is pushed outside the market. Employer 1 favors the type 0 agent, employer 3, the agent of type 1. Labor market equilibrium is the characterized by

$$\mathbf{w} = (w_1, w_3) = \left(\frac{11}{10} + \varepsilon; \frac{23}{30} + \varepsilon\right) \quad \text{where } \varepsilon > 0, \varepsilon \to 0$$

As in equilibrium 1, jobs 1 and 3 are respectively ...lled by agents of type 0 and 1. Let us compare the impact of competition on the compensation of each job - and then of each agent type. The pay in job 1 is increased by almost 10%, that of job 3 almost 15%.

The case 2   involves $U_3(S_3;1) > U_1(S_1;1) > U_2(S_2;1)$ but $U_1(S_1;0) > U_3(S_3;0) > U_2(S_2;0)$. for $U_{(0)} = U_2(S_2;0) = \frac{73}{40}$ and $U_{(1)} = U_2(S_2;1) = \frac{159}{80}$ then $0 \bar{A}_1 1$, as a matter of facts, $w_1\left(U_{(0)};0\right) = \frac{53}{40} < \frac{119}{80} = w_1\left(U_{(1)};1\right)$, and $0 \acute{A}_3 1$ since $w_3\left(U_{(0)};0\right) = \frac{73}{60} > \frac{119}{120} = w_3\left(U_{(1)};1\right)$. Employers 1 and 3 do not compete with each others: for all θ, $U_\theta = U_{(\theta)} + \varepsilon$.

218

Equilibrium 3.2 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = \left(\frac{31}{20}, \frac{93}{40}, \frac{4}{3}\right)$ employer 2 is pushed outside the market. Employer 1 favors the agent of type 0, employer 3, the agent of type 1. Labor market is then characterized by

$$\mathbf{w} = (w_1, w_3) = \left(\frac{53}{40} + \varepsilon; \frac{119}{120} + \varepsilon\right) \quad \text{where } \varepsilon > 0, \varepsilon \to 0$$

Previous observation as regards the relative impact of competition on each job compensation is con...rmed here.

The case 3 involves $U_3(S_3; 1) > U_2(S_2; 1) > U_1(S_1; 1)$ but $U_1(S_1; 0) > U_3(S_3; 0) > U_2(S_2; 0)$. Let us recall that the ...rst concern of an employer is to make a positive pro...t. Employer 1 can at least impose herself in the competition for type 0 employees, employer 3 in the competition for the agent of type 1. Employer 2, once again, will be pushed outside the market. For $U_{(0)} = U_2(S_2; 0) = \frac{79}{40}$ and $U_{(1)} = U_2(S_2; 1) = \frac{177}{80}$, $0 \bar{A}_1 1$, as a matter of facts, $w_1\left(U_{(0)}; 0\right) = \frac{59}{40} < \frac{137}{80} = w_1\left(U_{(1)}; 1\right)$, and $0 \bar{A}_3 1$ since $w_3\left(U_{(0)}; 0\right) = \frac{79}{60} > \frac{137}{120} = w_3\left(U_{(1)}; 1\right)$. Employers 1 and 3 do not directly compete: for all θ, $U_\theta = U_{(\theta)} + \varepsilon$.

Equilibrium 3.3 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = \left(\frac{33}{20}, \frac{99}{40}, \frac{4}{3}\right)$ employer 2 is pushed outside the market. Employer 1 favors the type 0 agent, employer 3, the type 1 applicant. Labor market is then characterized by

$$\mathbf{w} = (w_1, w_3) = \left(\frac{59}{40} + \varepsilon; \frac{137}{120} + \varepsilon\right) \quad \text{where } \varepsilon > 0, \varepsilon \to 0$$

Previous observations can be renewed here.

The case 4 involves $U_3(S_3; 1) > U_2(S_2; 1) > U_1(S_1; 1)$ but $U_1(S_1; 0) > U_2(S_2; 0) > U_3(S_3; 0)$. Firm 2 remains outside the labor market. Its potential entry imposes ...rms 1 and 3 reservation utilities $U_2(S_2; 0) = \frac{41}{20}$ and $U_2(S_2; 1) = \frac{93}{40}$. Though, we have $w_1(U_2(S_2; 0); 0) = \frac{31}{20} < w_1(U_2(S_2; 1); 1) = \frac{73}{40}$ i.e. ...rm 1 prefers agents of type 0; and

219

$w_3 \left(U_2 \left(S_2; 0 \right); 0 \right) = \frac{41}{30} > w_3 \left(U_2 \left(S_2; 1 \right); 1 \right) = \frac{73}{60}$ i.e. ...rm 3 prefers agents of type 1. Firms 1 and 3 do not directly compete with each other.

Equilibrium 3.4 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) = \left(\frac{34}{20}, \frac{102}{40}, \frac{4}{3} \right)$ employer 2 is pushed aside the market. Employer 1 favors type 0 agent, employer 3, applicant of type 1. Labor market is characterized by

$$\mathbf{w} = (w_1, w_3) = \left(\frac{31}{20} + \varepsilon; \frac{73}{60} + \varepsilon \right) \quad \text{where } \varepsilon > 0, \ \varepsilon \to 0$$

All the cases above represent a polarized competition: the point is just about neutralizing employer 2 but ...rms 1 and 3 do not exert any pressure toward each others. Cases 5 and 6 involve multipolar competition.

Multipolar competition: cases 5 and 6. The case 5 involves $U_2 \left(S_2; 1 \right) > U_3 \left(S_3; 1 \right) > U_1 \left(S_1; 1 \right)$ but $U_1 \left(S_1; 0 \right) > U_2 \left(S_2; 0 \right) > U_3 \left(S_3; 0 \right)$. This is then employer 3 who is pushed aside from labor market. For $U_{(0)} = U_3 \left(S_3; 0 \right) = 2$ and $U_{(1)} = U_3 \left(S_3; 1 \right) = \frac{5}{2}$, $0 \hat{A}_1 1$ - since $w_1 \left(U_{(0)}; 0 \right) = \frac{3}{2} < 2 = w_1 \left(U_{(1)}; 1 \right)$, but also $0 \hat{A}_2 1$ - since $w_2 \left(U_{(0)}; 0 \right) = \frac{5}{2} < \frac{8}{3} = w_2 \left(U_{(1)}; 1 \right)$. Employers 1 and 2 compete directly to obtain the agent of type 0. Employer 2 is prepared to o¤er up to $\frac{8}{3}$ to obtain this agent which corresponds to utility $U_2 \left(\frac{8}{3}; 0 \right) = \frac{13}{6}$, while the maximal wage o¤er of employer 1 is 2 which corresponds to utility $\frac{5}{2} > \frac{13}{6}$. Employer 1 prevails. It is enough to o¤er wage $w_1 = \min \left\{ \frac{2}{3} \left(U_2 \left(\frac{8}{3}; 0 \right) + \varepsilon \right) + \frac{1}{3} (2 \ 0), U_2 \left(\frac{8}{3}; 0 \right) + \varepsilon \ \frac{1}{2} \right\} = \frac{5}{3} + \varepsilon$. Employer 2's fallback position it to hire a type 1 agent to which she o¤ers $w_2 = \frac{8}{3} + \varepsilon$. Hence, $U_1 = U_{(1)} + \varepsilon$ but $U_0 = U_2 \left(w_2 \left(U_{(1)}; 1 \right); 0 \right) + \varepsilon > U_{(0)} + \varepsilon$.

The case 6 involves $U_2 \left(S_2; 1 \right) > U_1 \left(S_1; 1 \right) > U_3 \left(S_3; 1 \right)$ but $U_2 \left(S_2; 0 \right) > U_1 \left(S_1; 0 \right) > U_3 \left(S_3; 0 \right)$. This change does not anything[15] to the case 5.

---

[15] Firm 3 is again excluded from the labor market and hence $U_3 \left(S_3; 0 \right) = 2$ and $U_3 \left(S_3; 1 \right) = \frac{5}{2}$. We get $w_1 \left(U_3 \left(S_3; 0 \right); 0 \right) = \frac{3}{2} < w_1 \left(U_3 \left(S_3; 1 \right); 1 \right) = 2$ and $w_2 \left(U_3 \left(S_3; 0 \right); 0 \right) = \frac{5}{2} < w_2 \left(U_3 \left(S_3; 1 \right); 1 \right) = \frac{8}{3}$ i.e. ...rms 1 and 2 compete to hire an agent of type 0. Maximal pay o¤ers of ...rms 1 and 2 to an agent of type 0 are respectively: 2 and $\frac{8}{3}$. These o¤ers provide to agent 0 utilities $U_1 (2; 0) = \frac{5}{2}$ and $U_2 \left(\frac{8}{3}; 0 \right) = \frac{13}{6} < \frac{5}{2}$. Then, it is the ...rm 1 who prevails, limiting her o¤er to $w_1 \left(U_2 \left(\frac{8}{3}; 0 \right) + \varepsilon; 0 \right)$.

Equilibria 3.5 and 3.6 For $I_B = \frac{3}{2}$, $n_0 = n_1 = 1$ employers 1, 2 and 3 compete. Assuming $(S_1, S_2, S_3) \in \left(\frac{36}{20}, \frac{108}{40}, \frac{4}{3}\right), \left(\frac{44}{20}, \frac{132}{40}, \frac{4}{3}\right)$ employer 3 pushed aside from the labor market. Employer 1 favors agents of type 0, employer 2, the agent of type 1. Labor market is characterized by

$$\mathbf{w} = (w_1, w_2) = \left(\frac{5}{3} + \varepsilon; \frac{8}{3} + \varepsilon\right) \quad \text{where } \varepsilon > 0, \ \varepsilon \to 0$$

5.5 Attempt of a general formulation of labor market equilibrium

In this last section, we propose a de...nition of the labor market equilibrium and show its existence. Such equilibrium must specify for each job: 1) whether it is ...lled or not, 2) if it is the type of the agent who hold it, 3) the level of its compensation.

5.5.1 The building of a best response function

Let us assume four jobs are available: $\left(S_j, \phi_j, \psi_j\right)$, $j \in \{1, 2, 3, 4\}$. Three agents search for a job: one with trait $\theta = 0$, and two with trait $\theta = 1$. Let us build the reaction function of ...rm j. Let $j, j', j'', j^{(3)} \in \{1, 2, 3, 4\}$ be such that $j \le j' \le j'' \le j^{(3)}$ while $U_{j'}(w_{j'}; 1) \ge U_{j''}(w_{j''}; 1) \ge U_{j^{(3)}}\left(w_{j^{(3)}}; 1\right)$.[16] This latter condition is about the indexes j'

---

[16] Nota:
$$U_{j'}(w_{j'}; 1) \ge U_{j''}(w_{j''}; 1); \quad U_j(w_{j'}; 0) \ge U_{j''}(w_{j''}; 0)$$

Take for instance the case $b'' > b'$, $b'' > a_1'' = a_0'' + \gamma_h$ and $a_1' = a_0' + \gamma_h > b' > a_0'$. then $\max\{a_1', b'\} \ge \max\{a_1'', b''\}$ but $\max\{a_0', b''\} \ge \max\{a_0', b'\}$.

to $j^{(3)}$: they adjust so that it keeps holding.[17]

$$r_j(\mathbf{w}_{\neg j}) = \begin{cases} \min\{w_j(I_B;0), w_j(I_B;1)\} = w_j(I_B;1) \\[2em] \min\{w_j(I_B;0), w_j(I_B;1)\} = w_j(I_B;1) \\[2em] \min\begin{cases} w_j(I_B;0) \\ w_j(U_{j''}(w_{j''};1)+\varepsilon;1) \end{cases} \\[2em] \min\begin{cases} w_j(I_B;0) \\ w_j(U_{j''}(w_{j''};1)+\varepsilon;1) \end{cases} \\[2em] \min\begin{cases} w_j\left(\max_{k\in\{j',j'',j^{(3)}\}} U_k(w_k;0)+\varepsilon;0\right) \\ w_j(U_{j''}(w_{j''};1)+\varepsilon;1) \end{cases} \\[2em] S_j \end{cases}$$

if $\cdot S_j$ and $\forall k \in \left(j',j'',j^{(3)}\right]^a$, $w_k(I_B;1) > w_k$.

if $\cdot S_j$, $w_{j'}(I_B;0) > w_{j'} , w_{j'}(I_B;1)$ $\forall k \in \left(j'',j^{(3)}\right], w_k(I_B;1) > w_k$.

if $\cdot S_j$, $\forall k \in \{j',j''\}$, $w_k(I_B;0) > w_k , w_k(I_B;1)$ but $w_{j^{(3)}}(I_B;1) > w_{j^{(3)}}$.

if $\cdot S_j$ and $\forall k \in \left(j',j'',j^{(3)}\right]^a$, $w_k(I_B;0) > w_k , w_k(I_B;1)$.

if $\cdot S_j$, $\forall k \in \left(j',j'',j^{(3)}\right]$, $w_k , w_k(I_B;1)$, but $w_k(I_B;0) ? w_k$.

otherwise

where $\varepsilon > 0, \varepsilon \to 0$. Put in a more compact writing

$$r_j(\mathbf{w}_{\neg j}) = \min\left\{\max\left\{w_j(I_B;0), w_j\left(\max_{k\in\{j',j'',j^{(3)}\}} U_k(w_k;0)+\varepsilon;0\right)\right\}\overset{S_j}{,} \max\{w_j(I_B;1), w_j(U_{j''}(w_{j''};1)+\varepsilon;1)\}\right\}$$

Let the equilibrium of the labor market $\mathbf{w} = (w_1, w_2, w_3, w_4)$ be such that 1 hires the agent with trait 0, 2 and 3 agents with trait 1 while job 4 remains un...lled. This

---

[17]Note that, for any k, the condition $w_k(I_B;0) > w_k , w_k(I_B;1)$ is equivalent to $U_k(w_k;1) , I_B >_i U_k(w_k;0)$ so that clauses expressed below are all consistent with $U_{j'}(w_{j'};1) , U_{j''}(w_{j''};1) , U_{j^{(3)}}(w_{j^{(3)}};1)$.

equilibrium is such that

$$
w_1 = \max \left\{ \begin{array}{l} w_1\left(I_B; 0\right) \\ w_1 \left[\max_{k \in \{2,3,4\}} U_k\left(w_k; 0\right) + \varepsilon; 0\right] \end{array} \right\} \cdot \max \left\{ \begin{array}{l} w_1\left(I_B; 1\right) \\ w_1 \left[\max_{k \in \{2,3,4\}} U_k\left(w_k; 1\right) + \varepsilon; 1\right] \end{array} \right\} ;
$$

$$
w_2 = \max \left\{ \begin{array}{l} w_2\left(I_B; 1\right) \\ w_2 \left[\max_{k \in \{1,4\}} U_k\left(w_k; 1\right) + \varepsilon; 1\right] \end{array} \right\} \cdot \max \left\{ \begin{array}{l} w_2\left(I_B; 0\right) \\ w_2 \left[\max_{k \in \{1,4\}} U_k\left(w_k; 0\right) + \varepsilon; 0\right] \end{array} \right\} ;
$$

$$
w_3 = \max \left\{ \begin{array}{l} w_3\left(I_B; 1\right) \\ w_3 \left[\max_{k \in \{1,4\}} U_k\left(w_k; 1\right) + \varepsilon; 1\right] \end{array} \right\} \cdot \max \left\{ \begin{array}{l} w_3\left(I_B; 0\right) \\ w_3 \left[\max_{k \in \{1,4\}} U_k\left(w_k; 0\right) + \varepsilon; 0\right] \end{array} \right\} ;
$$

$$
w_4 = S_4 \cdot \max \left\{ \begin{array}{l} w_4\left(I_B; 0\right) \\ w_4 \left[\max_{k \in \{1,2,3\}} U_k\left(w_k; 0\right) + \varepsilon; 0\right] \end{array} \right\} , S_4 \cdot \max \left\{ \begin{array}{l} w_4\left(I_B; 1\right) \\ w_4 \left[\max_{k \in \{1,2,3\}} U_k\left(w_k; 1\right) + \right. \end{array} \right.
$$

where for all $k \in \{1, 2, 3\}$, $w_k \cdot S_k$. Note that $U_2\left(w_2; 1\right) = U_3\left(w_3; 1\right)$.

5.5.2 Labor market equilibrium

For any vector $\mathbf{w}_{-j} = \left(w_k\right)_{k \in J - \{j\}}$, for all $\theta \in \{0, 1\}$, let $k_\theta\left(j; \mathbf{w}_{-j}\right) \in J - \{j\}$ index any
...rm such that

$$
\begin{array}{l}
\# \left\{ k \in J - \{j\} | U_k\left(w_k; \theta\right) > U_{k_\theta\left(j; \mathbf{w}_{-j}\right)} \left[w_{k_\theta\left(j; \mathbf{w}_{-j}\right)}; \theta\right] \right\}^a < n_\theta \\
\# \left\{ k \in J - \{j\} | U_k\left(w_k; \theta\right) , U_{k_\theta\left(j; \mathbf{w}_{-j}\right)} \left[w_{k_\theta\left(j; \mathbf{w}_{-j}\right)}; \theta\right] \right\}^a , n_\theta
\end{array}
$$

Whatever $\mathbf{w}_{-j}$ and $\theta \in \{0, 1\}$, ...rms belonging to $J - \{j\}$ can be ranked by decreasing $U\left(w; \theta\right)$. Firm $k_\theta\left(j; \mathbf{w}_{-j}\right)$'s hiring o¤er simply represents the n_θ-teenth best o¤er (that of ...rm j being excluded) made to agent with trait θ.

One can check that, in the equilibrium described above, $k_0\left(1; \mathbf{w}_{-1}\right) \in \arg\max_{k \in \{2,3,4\}} U_k\left(w_k; 0\right)$, $k_0\left(2; \mathbf{w}_{-2}\right) = k_0\left(3; \mathbf{w}_{-3}\right) = k_0\left(4; \mathbf{w}_{-4}\right) = 1$ while $k_1\left(2; \mathbf{w}_{-2}\right) = k_1\left(3; \mathbf{w}_{-3}\right) \in \arg\max_{k \in \{1,4\}} U_k\left(w_k; 1\right)$ and $k_1\left(1; \mathbf{w}_{-1}\right) = k_1\left(4; \mathbf{w}_{-4}\right) \in \{2, 3\}$.

Assuming it is pro...table for j to hire an agent with trait θ, her o¤er has just to be strictly preferred to that of $k_\theta\left(j; \mathbf{w}_{-j}\right)$ to attract an agent with trait θ. Given $\mathbf{w}_{-j}$ ...rm

223

j's reaction function[18] $r_j(.)$ is given by

$$r_j(\mathbf{w}_{\neg j}) = \min \begin{Bmatrix} \max \left\{ w_j(I_B; 0), w_j \left(U_{k_0(j; \mathbf{w}_{\neg j})} - w_{k_0(j; \mathbf{w}_{\neg j})}; 0 \right) + \varepsilon; 0 \right\} \\ \max \left\{ w_j(I_B; 1), w_j \left(U_{k_1(j; \mathbf{w}_{\neg j})} - w_{k_1(j; \mathbf{w}_{\neg j})}; 1 \right) + \varepsilon; 1 \right\} \end{Bmatrix}, S_j$$

where $\varepsilon > 0, \varepsilon \to 0$.

Note that, since a job left closed entails a null pro...t, it is equivalent to assume that corresponding employer spends S_j to obtain surplus S_j and hence $r_j(\mathbf{w}_{\neg j}) = S_j$ when minimal required transfer is strictly higher than S_j.[19]

De...nition An equilibrium of the labor market is a vector w such that for all $j \in J$: $w_j \in r_j(\mathbf{w}_{\neg j})$.

We now show this equilibrium exists.

Proposition 7 If, for all $\theta \in \{0, 1\}$, $n_\theta \geq 1$, labor market equilibrium exists.

Proof 7 We follow the standard Nash equilibrium existence proof.[20] For all $j \in J$, let us de...ne $\rho_j : \mathfrak{E}_{k \in J}[0, S_k] \to [0, S_j]$ by $\rho_j(w) = r_j(\mathbf{w}_{\neg j})$. De...ne the correspondence $\rho : \mathfrak{E}_{j \in J}[0, S_j] \rightrightarrows \mathfrak{E}_{j \in J}[0, S_j]$ to be the cartesian product of the ρ_j. A ...xed point of ρ is a w such that $w \in \rho(w)$, so that, for each ...rm, $w_j \in \rho_j(w)$. Thus a ...xed point of ρ is a labor market equilibrium. From Kakutani's theorem the following are su¢cient conditions for ρ to have a ...xed point: (i) $\mathfrak{E}_{j \in J}[0, S_j]$ is compact, convex, nonempty subset of a (...nite-dimensional) euclidean space; (ii) $\rho(w)$ is nonempty for all w; (iii) $\rho(w)$ is convex for all w; (iv) $\rho(.)$ has a closed graph.

Since $\forall j \in J, S_j \geq 0, \mathfrak{E}_{j \in J}[0, S_j]$ is indeed compact, convex and nonempty; furthermore it is a subset of $\mathbb{R}^J$ with $J = \#J < \infty$ so that (i) is clearly satis...ed. For all $\theta \in \{0, 1\}$, assuming $n_\theta \geq 1, \forall j \in J, \forall w_{\neg j} \in \mathfrak{E}_{k \in J \neg \{j\}}[0, S_k]$, $k_\theta(j; \mathbf{w}_{\neg j})$ exists. Hence, $\forall \theta \in \{0, 1\}, U_{k_\theta(j; \mathbf{w}_{\neg j})} - w_{k_\theta(j; \mathbf{w}_{\neg j})}; \theta$ is well de...ned and $\forall j \in J, \forall w_{\neg j} \in \mathfrak{E}_{k \in J \neg \{j\}}[0, S_k]$,

---

[18]Maybe this function should be understood as capturing an algorithm leading to labor market equilibrium rather than as a reaction function.

[19]One can interpret this as re‡ecting the assumption that the employer choose to do the job himself.

[20]See for instance Fudenberg & Tirole (1996, chapter 1, p.29).

$r_j(\mathbf{w}_{-j})$, and hence $\rho_j(\mathbf{w})$, is nonempty which entails (ii). If $\rho(\mathbf{w})$ were not convex, there would exist $w^0 \in \rho(\mathbf{w})$ and $w^{00} \in \rho(\mathbf{w})$, and $\lambda \in]0,1[$ such that $\lambda w^0 + (1-\lambda)w^{00} \notin \rho(\mathbf{w})$. For all ...rm j,

$$S_j\left(\lambda w_j^0 + (1-\lambda)w_j^{00}\right) = \lambda S_j\left(w_j^0\right) + (1-\lambda)S_j\left(w_j^{00}\right)$$

so that if both w_j^0 and w_j^{00} are best responses to w_{-j}, then so is their weighted average. This veri...es (iii).

Assume that (iv) is violated so there is a sequence $(\mathbf{w}^n, \hat{\mathbf{w}}^n) \to (\mathbf{w}, \hat{\mathbf{w}})$ such that $\hat{\mathbf{w}}^n \in \rho(\mathbf{w}^n)$ but $\hat{\mathbf{w}} \notin \rho(\mathbf{w})$. Then $\hat{w}_j \notin \rho_j(\mathbf{w})$ for some j. Thus there is an $\epsilon > 0$ and a w_j^0 such that $S_j\left(w_j^0\right) > S_j\left(\hat{w}_j\right) + 3\epsilon$. Since $(\mathbf{w}^n, \hat{\mathbf{w}}^n) \to (\mathbf{w}, \hat{\mathbf{w}})$, for n su¢ciently large

$$S_j\left(w_j^0\right) > S_j\left(w_j^0\right) - \epsilon > S_j\left(\hat{w}_j\right) + 2\epsilon > S_j\left(\hat{w}_j^n\right) + \epsilon$$

i.e. w_j^0 does strictly better against w_{-j}^n than $\hat{w}_j^n$ does, which contradicts $\hat{w}_j^n \in \rho_j(\mathbf{w}^n)$. This veri...es (iv).

As compensating di¤erentials claims, it must be possible to show that in the equilibrium, two agents of a given type should bene...t the same utility level. The case of no shortage of type 1 agents does not raise particular obstacles: all the agents have for reservation utility I_B, and all the ...rms succeed in ...lling their job - at least those satisfying minimal pro...tability conditions.[21] The di¢culties we meet in other cases echo the multipolar characteristics of competition. Each ...rms has to control: for potential entries, for the pressure exerted by insiders willing to hire an agent of another type. Our attempts to treat these problems are avaible upon request.

---

[21] That is such that $S \geq w(I_B; 1)$.